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P-vortices, gauge copies and lattice size

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Abstract: We study the dependence of the center-projected string tension on both the lattice size, and the number of gauge copies used for maximal center gauge fixing. We show that a recent finding of Bornyakov, Komarov, Polikarpov, and Veselov (hep-lat/0002017), indicating a substantial breakdown of center dominance in maximal center gauge, is only obtained for rather small lattice volumes, and is not found in numerical simulations on larger lattices. It is shown that center-projected Creutz ratios approach the full asymptotic string tension as lattice size increases, and that the P-vortex density is consistent, at moderately weak couplings, with 2-loop scaling behavior.

Keywords: Solitons Monopoles and Instantons, Confinement, Lattice Gauge Field Theories.

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Contents

1. Introduction

In the last few years there has been renewed interest, within the lattice gauge theory community, in the center vortex theory of confinement [1]. The revival of this old idea is due to a number of numerical studies, which all indicate that center vortices are ubiquitous in the QCD vacuum and give rise to the linear confining potential [2]–[15]. Recently, however, a paper by Bornyakov, Komarov, Polikarpov, and Veselov (BKPV) [16] appeared which questions the validity of a procedure, known as center projection in maximal center gauge, which was essential in many of these studies. The claim is that when large numbers of random gauge copies are used in fixing the gauge (in an effort to minimize the Gribov copy problem), the center-projected string tension underestimates the full string tension by as much as 30%. Our purpose in the present article is to show that, while BKPV have certainly raised an important issue, their actual data was strongly affected by the rather small lattice volumes used in the numerical simulations. We will show that the conclusions drawn from this data are not sustained by simulations on larger lattices, which generally agree with our previously reported results [4].

In the direct version of maximal center gauge [4], the procedure is to maximize

$$R = \sum_{\mu} \sum_{x} \mathrm{Tr}_A [U_\mu(x)] \tag{1.1}$$

by an iterative over-relaxation procedure, where $\mathrm{Tr}_A[U]$ is the trace of U in the adjoint representation. Let R_n denote the value of R after n over-relaxation sweeps. When R_n is judged to have converged, e.g. according to a criterion of the form

$$\frac{R_n - R_{n-50}}{R_n} < \delta \tag{1.2}$$

then the link variables $U_\mu(x)$ are projected onto the nearest center element Z. In SU(2) lattice gauge theory, the projection is simply

$$Z_\mu(x) = \mathrm{sign}\,\mathrm{Tr}[U_\mu(x)]\,. \tag{1.3}$$

Center-projected Wilson loops, Creutz ratios, etc. are observables computed from the center-projected link variables.

Finding a gauge transformation g which, when operating on a given lattice configuration U, produces a configuration ${}^gU$ which maximizes R in eq. (1.1), is equivalent to the problem of finding the zero-temperature ground state of a spin system with action

$$S = -\sum_{x,\mu} \mathrm{Tr}_A[g(x)U_\mu(x)g^\dagger(x+\hat{\mu})]\,. \tag{1.4}$$

Here $g(x)$ regarded as the spin-field, and $U_\mu(x)$ a fixed background. There is no known method for finding the exact ground state of this system for arbitrary $U_\mu(x)$. The technique which has been used until now to generate a configuration ${}^gU$, which is subsequently center-projected, is the over-relaxation method. This technique can be regarded as a rapid cooling of the analog spin system, starting from the configuration $g = I$. The result of this rapid cooling is not the true ground state of the system, but rather a kind of "super-cooled" metastable state. Nevertheless, center projection based on these super-cooled states has been successful in locating center vortices on unprojected lattices, where the evidence of this success is as follows (c.f. e.g. [4, 6]):

- Thin vortex excitations on the projected lattice, known as "P-vortices", are closely correlated with the sign of the trace of Wilson loops on the unprojected lattice. In particular, if $W_n(C)$ denotes the VEV of a subensemble of Wilson loops whose minimal area is pierced by n P-vortices, then $W_n(C)/W_0(C) \to (-1)^n$ in the limit of large loop size. This is the expected behavior if P-vortices, on the projected lattice, are located roughly in the middle of thick center vortices on the unprojected lattice.

- Projected Creutz ratios $\chi_{cp}(I, I)$ are close to the asymptotic string tension on the unprojected lattice, a property known as "center dominance". This property indicates that center vortices are responsible for the entire asymptotic string tension.

- The density of P-vortices, from $\beta = 2.3$ onward, scales according to asymptotic freedom.

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- Removing center vortices (located via the projected lattice) from unprojected lattices also removes confinement and chiral symmetry breaking, and brings the topological charge on the lattice to zero.

By making random gauge copies of the background lattice field U, and carrying out over-relaxation on each copy, we obtain a set of super-cooled spin-field states $\{g\}$, and a corresponding set of "Gribov copies" $\{^gU\}$. It is not strictly necessary that the supercooled states should be physically equivalent to the true ground state of the analog spin system, nor, as a consequence, that center projection on any of the Gribov copies $\{^gU\}$ should be equivalent to center projection of the true maximum of (1.1). In fact, there exists some evidence against this equivalence [18, 17]. It *is* necessary, however, that the over-relaxation/projection procedure, which we have advocated for identifying vortices on the lattice, is well defined and unambiguous. In particular, it is required that either all Gribov copies $\{^gU\}$ generated by the over-relaxation cooling procedure are equivalent under center projection, or else that there is a definite criterion for selecting the best copy out of a large set.

In ref. [4] it was found that projected Wilson loops (and therefore P-vortex locations) are highly correlated, although not identical, among Gribov copies. The obvious criterion for choosing the "best" Gribov copy in a set is to choose the copy which maximizes the gauge-fixing functional R. Although we found some small dependence of projected observables on the number of gauge copies generated (we picked the best of three copies in our original work [4]), this copy-number dependence did not appear to be a large effect. The recent work of BKPV claims that, on the contrary, there is a very significant dependence of the projected string tension on the number of gauge copies used, and in the large copy-number limit there is a drastic breakdown of both center dominance and scaling.

Motivated by this claim, we have studied the sensitivity of center-projected Creutz ratios and the P-vortex density with respect to: (i) the number N_{copy} of random gauge copies used for maximal center gauge fixing; (ii) the lattice size; and (iii) the convergence parameter δ in eq. (1.2). Our results are presented below. We will here only discuss center projection in direct maximal center gauge, since this is the case treated in ref. [16], and it is also the center gauge with which we have the most experience. It should be noted, however, that alternatives to maximal center gauge do exist; laplacian center gauge [7], in particular, is free from the Gribov copy problem, and therefore immune to the BKPV criticism.

2. N_{copy} and lattice size dependence

We begin with the N_{copy} dependence, whose importance was recently emphasized by BKPV [16]. As noted above, when the over-relaxation gauge-fixing procedure is applied to different gauge copies of a given lattice configuration, slightly different values of R are obtained. One possible way to minimize the gauge copy dependence is to carry out the over-relaxation procedure on a number $N_{\rm copy}$ of random gauge copies, perform center projection on the copy with the largest value of R, and evaluate observables. Data obtained in this way, over a range of $N_{\rm copy}$ values, can then be extrapolated to the $N_{\rm copy} \to \infty$ limit.[1] In the original simulations of ref. [4], only three gauge copies were used, and there was no attempt to extrapolate to $N_{\rm copy} \to \infty$. Projected Creutz ratios $\chi_{cp}(I,I)$ were found to be close to the asymptotic string tensions reported by Bali et al. [19], for all $I \geq 2$. BKPV, however, calculate Creutz ratios in the range $N_{\rm copy} \in [1, 20]$, and extrapolate to $N_{\rm copy} \to \infty$ by fitting their data to the functional form

$$\chi_{\rm cp}^{N_{\rm copy}}(I,I) = \chi_{cp}(I,I) + \frac{c(I,I)}{\sqrt{N_{\rm copy}}} \,. \tag{2.1}$$

The result reported by BKPV is that projected string tensions, at $\beta = 2.4, 2.5$, underestimate the full string tension by about 20% at $N_{\rm copy} = 20$, and by as much as 30% in the extrapolation to $N_{\rm copy} \to \infty$. This result suggests that the center dominance previously reported in maximal center gauge was a numerical accident, a result of using too few gauge copies.

However, apart from the number of gauge copies, there is one other notable difference between the BKPV simulations and previous work. This is the matter of lattice size. Data reported in ref. [4] was obtained at $\beta = 2.3$ and $\beta = 2.4$ on 16^4 lattices, and at $\beta = 2.5$ on a 22^4 lattice. BKPV, on the other hand, used only 12^4 lattices at $\beta = 2.3$ and $\beta = 2.4$, and lattice size 16^4 at $\beta = 2.5$. This raises the question of whether the BKPV results, obtained on the smaller lattices, were seriously contaminated by finite-size effects. To find out, we have repeated the center-projection calculation at $\beta = 2.3$ and $\beta = 2.5$ on a variety of lattice sizes, for $N_{\rm copy} \in [1, 20]$. For the convergence parameter in eq. (1.2), we have used $\delta = 2 \times 10^{-7}$.

In figure 1 we display results for the Creutz ratio $\chi_{cp}^{N_{\rm copy}}(4,4)$ vs. $N_{\rm copy}$ at $\beta = 2.5$, for lattice sizes ranging from 8^4 to 28^4. Two features of this data are immediately apparent. First, there is indeed a slow downward trend in the Creutz ratio as $N_{\rm copy}$ increases, as noted by BKPV, but this effect is much more pronounced on smaller lattices than on larger lattices. Second, although Creutz ratios on the smaller lattices grossly underestimate the full string tension, the data appears to steadily increase towards the full asymptotic string tension, reported by Bali et al. [19], as the lattice

[1] P-vortex positions extracted from different Gribov copies are closely correlated, as one would expect if they locate physical objects [4]. It is possible, however, by choosing a very special starting configuration with links gauge-fixed to have mainly positive trace, to destroy the vortex finding property of center projection even for thin vortices [18, 17]. This is one aspect of the Gribov copy problem in maximal center gauge. Since the vortex-finding property is essential to center projection, it is evident that any unusual variation of the iterative gauge-fixing procedure, which destroys this property, must be avoided.

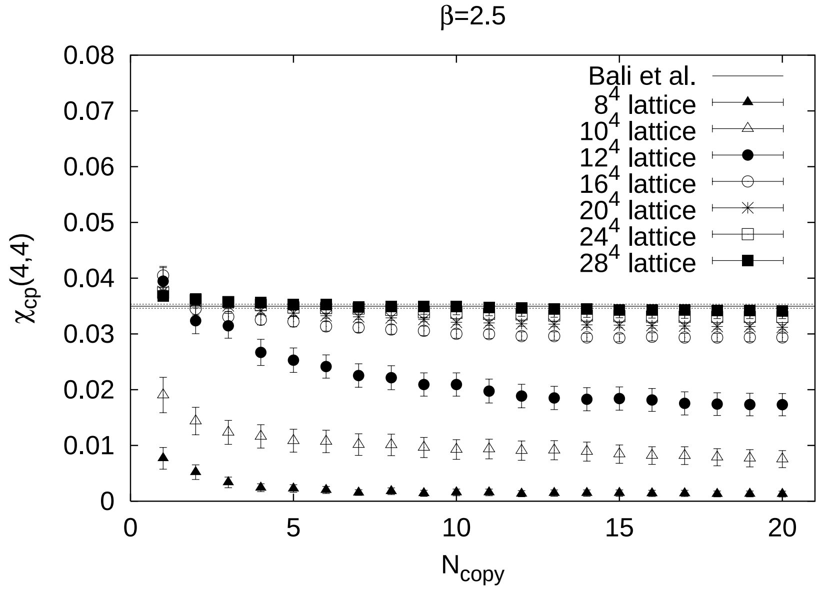

Figure 1: Center projected Creutz ratios $\chi_{cp}^{N_{\text{copy}}}(4,4)$ vs. N_{copy} at various lattice volumes, for $\beta = 2.5$. The solid line in this figure, and in figures 2–6, indicates the asymptotic string tension extracted by standard methods on unprojected lattices, reported by Bali et al. [19]. Dashed lines indicate the errorbar in this asymptotic string tension.

size increases. These trends in the data are by no means unique to the particular Creutz ratio $\chi_{cp}(4,4)$ at $\beta = 2.5$, but are typical of all of our results. For completeness we display, in figures 2 and 3, some other projected Creutz ratios $\chi_{cp}^{N_{\text{copy}}}(I,I)$ for I in the range $I = 2-5$, at couplings $\beta = 2.3$ and $\beta = 2.5$. In figures 2–6, solid (dashed) lines indicate the value (errorbar) of the asymptotic string tension on the unprojected lattice, reported in ref. [19].

In figure 4 we show the projected Creutz ratios $\chi_{cp}^{N_{\text{copy}}}(I,I)$ for $N_{\text{copy}} = 5, 10, 15, 20$ on the largest lattices we have used: 20^4 at $\beta = 2.3$ and $\beta = 2.4$, and 28^4 at $\beta = 2.5$. We also show the values of these Creutz ratios extrapolated to the $N_{\text{copy}} \to \infty$ limit, using the fitting function (2.1) suggested by BKPV.[2] As usual in maximal center gauge, all the $\chi_{cp}(I,I)$ for $I \geq 2$ are close to the asymptotic string tension, and these latest results are not far from our earlier results reported in ref. [4]. As in those earlier results, there appears to be a very gradual rise in $\chi_{cp}^{N_{\text{copy}}}(I,I)$ with I. It is not clear to us whether or not this latter tendency is very significant. The rise in Creutz ratios tends to be somewhat offset by the drop with increasing N_{copy}; in fact, there are signs that the $\chi_{cp}^{20}(I,I)$ (and extrapolated $\chi_{cp}^{\infty}(I,I)$) data at $\beta = 2.5$

[2] We note, however, that on the largest lattices our data varies so slowly with N_{copy} that it is impossible to strongly support this particular fitting function. Other fitting functions can be found which work about as well.

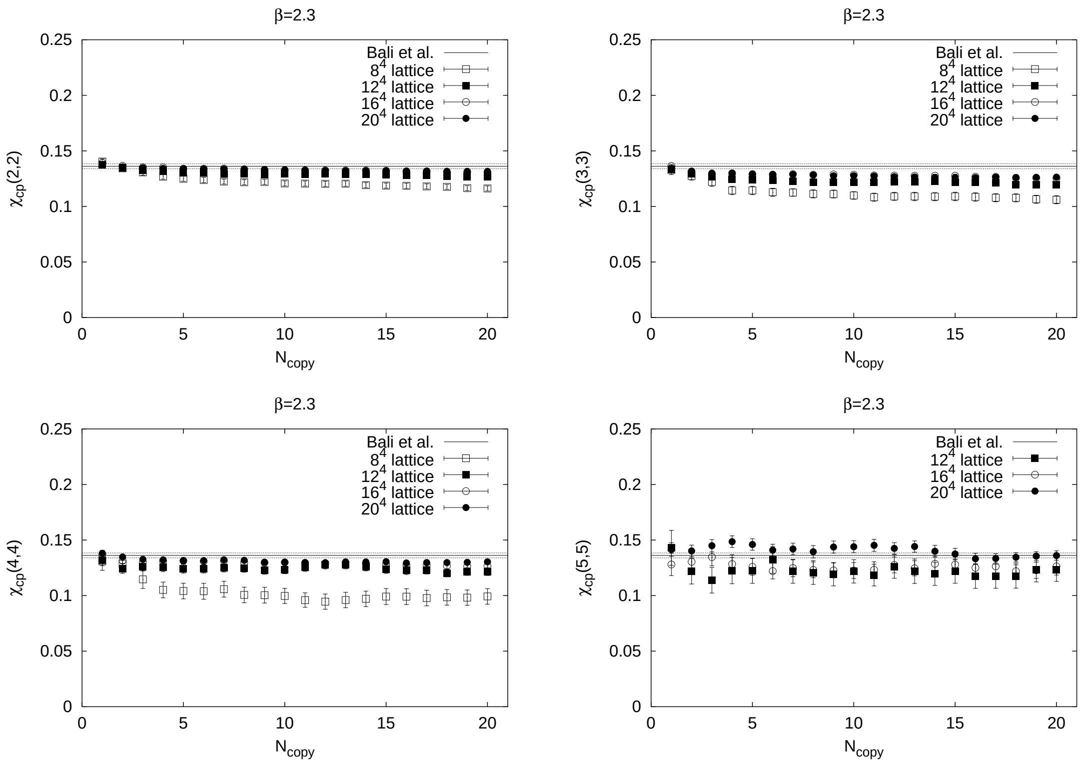

Figure 2: Center projected Creutz ratios $\chi_{cp}^{N_{\rm copy}}(I,I)$ vs. $N_{\rm copy}$ and lattice volume, at $\beta = 2.3$ and $I = 2 - 5$.

is leveling off at the largest I values, on a 28^4 lattice. These indications, however, are based on some of our least reliable data points. With sufficient computer time, the "rising" issue could presumably be settled, but this will have to be left for future investigation.

As another way of showing lattice size dependence, we take the average of the projected ($N_{\rm copy} \to \infty$) Creutz ratios $\chi_{cp}(I,I)$ in the range $I = 2 - 5$

$$\chi_{av} \equiv \frac{1}{4} \sum_{I=2}^{5} \chi_{cp}(I,I)\,. \tag{2.2}$$

Figure 5 shows the lattice size dependence of χ_{av}. Note the approach of χ_{av} to the full asymptotic string tension ($\sigma_{\rm SU(2)} = 0.035$ at $\beta = 2.5$) as lattice size increases.

2.1 Gauge-fixing convergence criterion

In addition to lattice volume and $N_{\rm copy}$ dependence, it is also worthwhile to check that the numerical results are stable when the gauge-fixing convergence criterion is strengthened, i.e. when the constant δ in eq. (1.2) is reduced. When δ is chosen too large, the center projected Creutz ratios come out significantly too high. This is illustrated in figure 6, which shows results for projected Creutz ratios with convergence

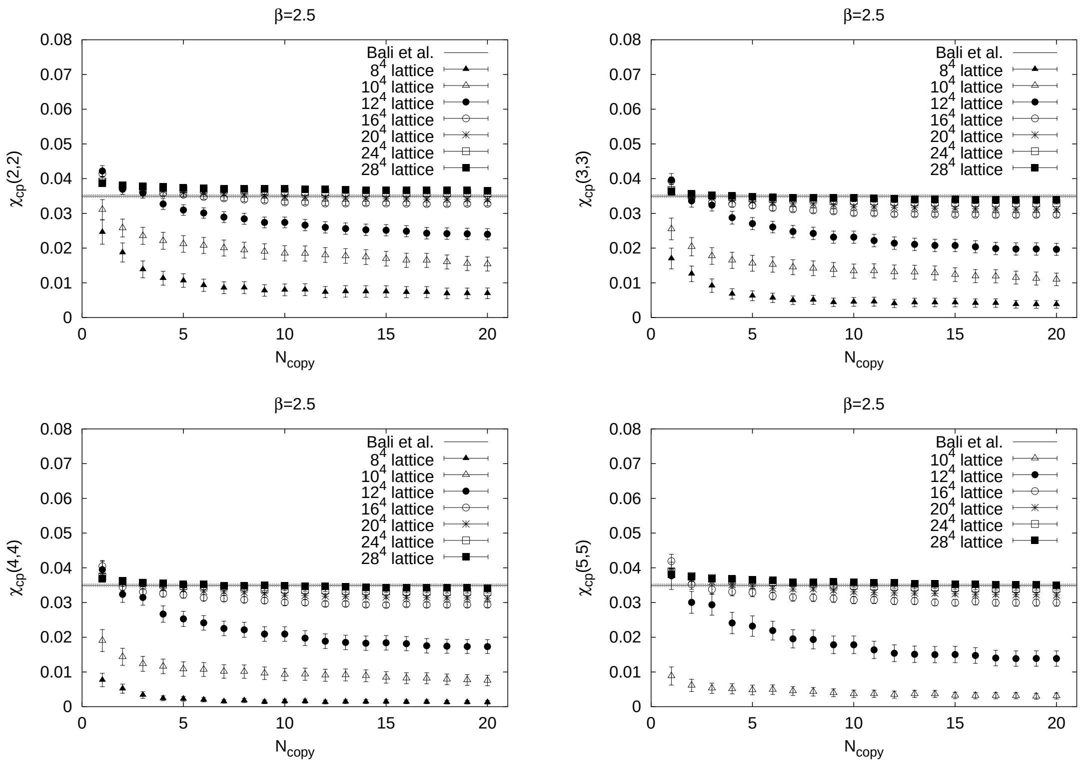

Figure 3: Center projected Creutz ratios $\chi_{cp}^{N_{\rm copy}}(I,I)$ vs. $N_{\rm copy}$ and lattice volume, at $\beta = 2.5$ and $I = 2-5$.

criteria $\delta = 10^{-2}, 10^{-3}, 10^{-4}, 2 \times 10^{-7}$ at $\beta = 2.5$ (24^4 lattice). The weakest convergence criterion, corresponding to $\delta = 10^{-2}$, is clearly insufficient for accurate results, but Creutz ratios obtained with the two smallest values of δ are fairly consistent, indicating that these numbers are not far from the $\delta \to 0$ limit.

3. Vortex density

Next we consider the scaling of the vortex density. The lattice P-vortex density p is the total number of P-vortex plaquettes (i.e. plaquettes on the projected lattice with $ZZZZ = -1$), divided by the total number of plaquettes on the lattice. This quantity is proportional to the average area taken up by P-vortices per unit lattice volume, and is determined from the center-projected plaquette expectation value

$$p = \frac{1}{2}(1 - W_{cp}[1,1]) \,. \tag{3.1}$$

If this quantity scales as predicted by asymptotic freedom, then we would have $p = p_{af}$, where

$$p_{af} = \frac{\rho}{6\Lambda^2} F(\beta) \,. \tag{3.2}$$

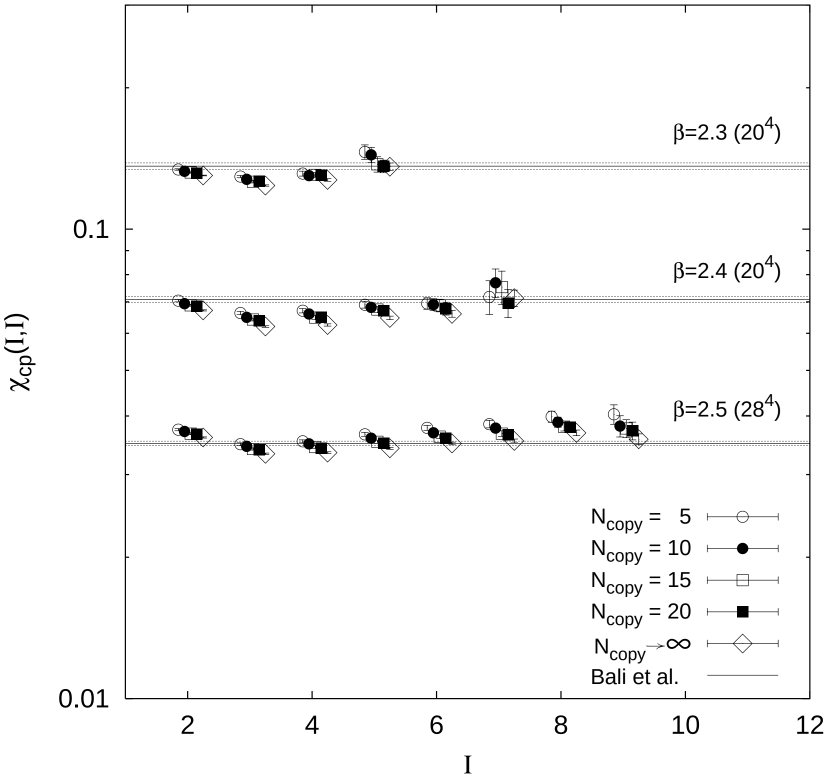

Figure 4: Center-projected Creutz ratios $\chi_{cp}(I, I)$ at $N_{\mathrm{copy}} = 5, 10, 15, 20$, and extrapolated to $N_{\mathrm{copy}} \to \infty$, on the largest lattices (20^4 lattices at $\beta = 2.3, 2.4$, and 28^4 at $\beta = 2.5$) used in our simulations.

Here ρ is the vortex density (average vortex area per unit volume) in physical units of inverse area, and

$$F(\beta) = \left(\frac{6\pi^2}{11}\beta\right)^{102/121} \exp\left[-\frac{6\pi^2}{11}\beta\right] . \tag{3.3}$$

In figure 7 we plot the lattice vortex density rescaled by the asymptotic freedom expression

$$\tilde{p} \equiv \frac{p}{F(\beta)} \tag{3.4}$$

which should be constant in the large β limit, if p scales according to asymptotic freedom. There appears to be good evidence for this kind of scaling, already for $\beta \geq 2.2$, in agreement with previous results [4,10]. In figure 7 and table 1, the vortex densities at $\beta = 2.3, 2.4, 2.5$ are taken from the largest lattices (20^4, 20^4 and 28^4, respectively), and extrapolated to the $N_{\mathrm{copy}} \to \infty$ limit by a fit to

$$p^{N_{\mathrm{copy}}} = p + \frac{c}{\sqrt{N_{\mathrm{copy}}}} . \tag{3.5}$$

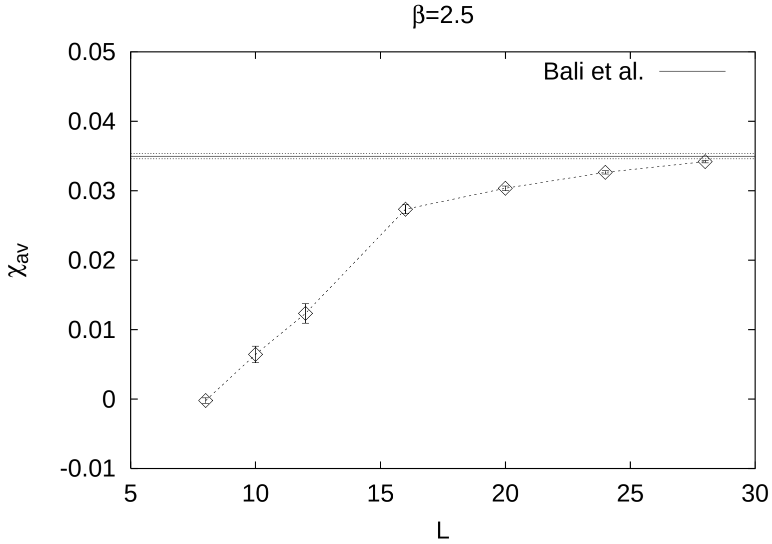

Figure 5: Average of projected, $N_{\rm copy} \to \infty$ extrapolated Creutz ratios $\chi_{cp}(I,I)$, in the range $I = 2-5$.

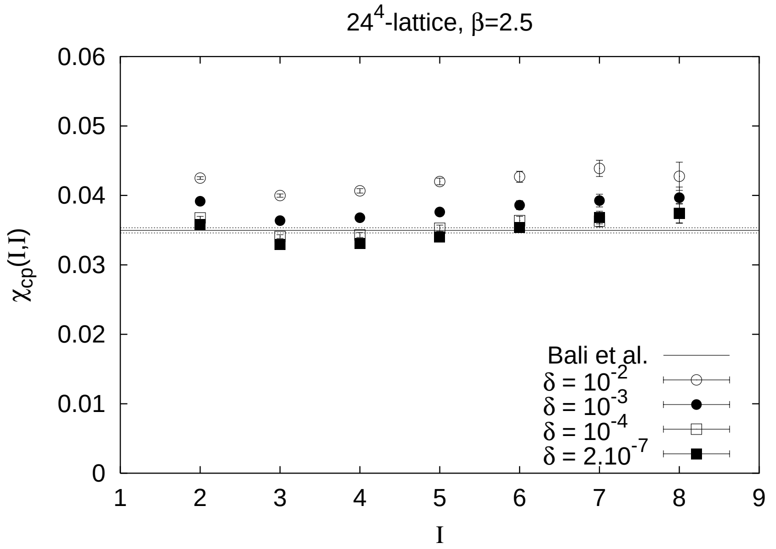

Figure 6: Effect of varying the convergence criterion δ on projected Creutz ratios (extrapolated to $N_{\rm copy} \to \infty$) at $\beta = 2.5$ on a 24^4 lattice.

The vortex densities at other values of β (with $N_{\rm copy} = 3$) are just taken from our previous work. For comparison, in table 1, we display our values for $\tilde{p}$, and the values of the rescaled asymptotic string tension

$$\tilde{\sigma} \equiv \frac{\sigma_{\rm SU(2)}}{F(\beta)}\,, \tag{3.6}$$

where $\sigma_{\rm SU(2)}$ is the string tension (in lattice units) on the unprojected lattice, at the given β value.

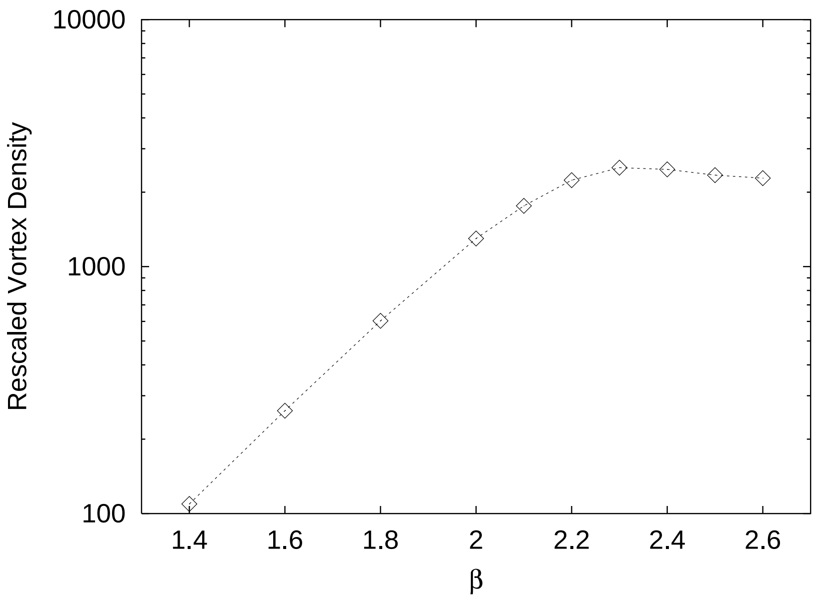

Figure 7: Rescaled vortex density $\tilde{p} = p/F(\beta)$, where p is the measured vortex density and $F(\beta)$ is the asymptotic freedom expression in eq. (3.3). Vortex density scales if $\tilde{p}$ is constant.

β	$\tilde{p}$	$\tilde{\sigma}$
2.3	$2.51(2) \times 10^3$	$3.89(6) \times 10^3$
2.4	$2.47(3) \times 10^3$	$3.34(5) \times 10^3$
2.5	$2.35(3) \times 10^3$	$2.73(3) \times 10^3$

Table 1: Rescaled vortex density $\tilde{p}$ and SU(2) string tension $\tilde{\sigma}$. These quantities should be constant in the scaling limit.

It is interesting to note that the scaling of vortex density p, in the range $\beta = 2.3 - 2.5$, is substantially better than the the scaling of the full asymptotic string tension $\sigma_{\mathrm{SU}(2)}$ in this range. Of course, all physical observables must exhibit the appropriate scaling as $\beta \to \infty$, but at intermediate β values some observables may satisfy scaling better than others. In particular, there is no requirement for the P-vortex density and the string tension to be exactly proportional at all β, and hence no reason for them to scale at intermediate β in exactly the same way. It is true that P-vortices give rise to the asymptotic string tension at any β, at least according to the center dominance hypothesis. However, the relationship between the P-vortex density, and the full asymptotic string tension, is not entirely direct. If there were no correlations at all between P-plaquettes in a plane, then the relationship would be simple at weak couplings: $\sigma = 2p$, where p is the fraction of plaquettes on the lattice that are P-plaquettes. However (as first noted by Engelhardt et al. [11]) there *are* short range correlations among P-plaquettes in a plane, and as a result the relation between σ and p may be fairly complicated. It is because of these correlations that σ and p need not vary with β in lockstep (except as $\beta \to \infty$), even if the entire string tension at any given β is entirely due to vortex effects.

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If the P-vortex density is physical, then it must scale eventually. The evidence that this density scales from about $\beta = 2.3$ onwards is therefore welcome, with the proviso, always present in numerical work, that this scaling does not break down at some still higher β values.

4. Vortex thickness

Why are such relatively large lattices required, in order to avoid large finite size effects in center projected Creutz ratios and vortex densities? We believe that the relevant length scale here is associated with the vortex thickness. Center vortices are surface-like objects, in $D = 4$ dimensions, which have a finite thickness in physical units. The thick vortex surface (or "core") bounds a Dirac 3-volume, which represents the region of discontinuity of a singular gauge transformation associated with the vortex. In ref. [17] we have explained the vortex-finding property of maximal center gauge in terms of the global properties of this singular gauge transformation. On a small lattice, however, with an extension comparable to the vortex thickness, these global aspects of the vortex field may be almost absent, and the minimal Dirac 3-volume could be quite small. In that case, the argument of ref. [17] breaks down. For this reason, we expect center projection to be less effective at finding vortices on small lattices, leading to underestimates of both the vortex density and the projected Creutz ratios.

Assuming there is some truth in this explanation, center projection can only be accurate for lattices whose extension is large compared to the vortex thickness. There are three independent ways of estimating the thickness of center vortices, which can be deduced from either

1. the ratio of "vortex-limited" Wilson loops [4];
2. the vortex free energy as a function of lattice size [14];
3. the adjoint string-breaking length [20].

Vortex-limited Wilson loops are defined in the following way: $W_n(C)$ is a Wilson loop evaluated on a sub-ensemble of unprojected configurations, selected so that precisely n P-vortices, in the corresponding center-projected configurations, pierce the minimal area of the loop. We can further make the restriction, for $W_1(C)$, that the negative P-vortex plaquette lies at (or touches) the center point of the rectangular loop. It is then expected that

$$\frac{W_1(C)}{W_0(C)} \longrightarrow -1 \tag{4.1}$$

in the limit where the vortex core is entirely contained within the loop (cf. ref. [4] for a more extended discussion). In figure 8 we show the data for W_1/W_0 vs. loop area

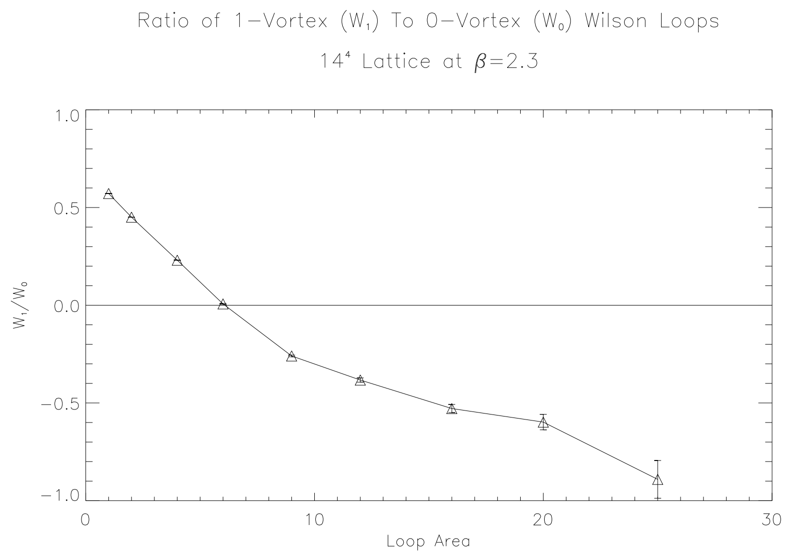

Figure 8: Ratio of the 1-Vortex to the 0-Vortex Wilson loops $W_1(C)/W_0(0)$, vs. loop area at $\beta = 2.3$.

at $\beta = 2.3$, taken from our previous work in ref. [4]. Judging from this figure, the vortex appears to almost fit inside a 5×5 loop, which leads to a rough estimate of the vortex radius, as it pierces a plane, of about 3 lattice spacings. At $\beta = 2.3$ we have $\sigma a^2 = 0.135$, and taking $\sigma = 5\,\text{fm}^{-2}$, the lattice spacing is $a = 0.164\,\text{fm}$. A diameter of 6 lattice spacings then corresponds to a vortex thickness of about one fermi.

A second estimate is obtained from the recent calculation of vortex free energy vs. lattice size, carried out numerically by Kovács and Tomboulis [14]. The vortex free energy is close to zero when the lattice extension is greater than the vortex thickness, and this again gives an estimate for the vortex thickness of a little over one fermi. Finally, if confinement is due to center vortices, then an $R \times T$ Wilson loop in the adjoint representation must change from a (Casimir scaling) area-law falloff to a (color-screening) perimeter-law falloff for charge separation R greater than the vortex thickness [21]. The adjoint string-breaking distance has been measured, by de Forcrand and Philipsen [20], to be $1.25\,\text{fm}$, and this distance provides us with a third estimate of the vortex thickness, which is roughly consistent with the other two.

At $\beta = 2.5$, one fermi corresponds to 12 lattice spacings. The lattice used by BKPV at $\beta = 2.5$ was only 16 lattice spacings across, and this may simply be inadequate for center projection to reliably identify vortices, in view of the above estimates for the vortex thickness. In fact, the P-vortex density is quite low on small lattices, increasing sharply up to $L = 16$ lattice spacings at $\beta = 2.5$, where it begins to level off. This is illustrated in figure 9; the data points are the extrapolated values for p at $N_{\text{copy}} \to \infty$.

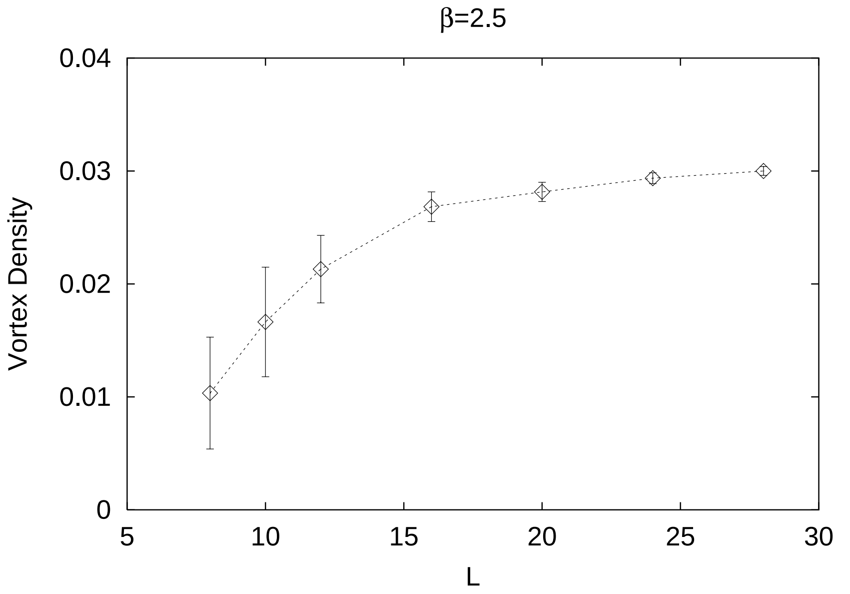

Figure 9: Vortex density p vs. the extension of L of the hypercubic lattice (volume L^4) at $\beta = 2.5$.

In order to estimate the average distance between vortices, we need to know the vortex density. The P-vortex density, discussed in the previous section, is in fact an overestimate of the actual center vortex density [10,12]. The reason is that P-vortices fluctuate within the thick vortex core, and P-vortex locations, while correlated among random Gribov copies, do vary somewhat from one random copy to another [4]. Thus, although P-vortices are certainly near the middle of center vortices, they are unlikely to be exactly in the middle. A more accurate estimate of the center vortex density is arrived at by either "smoothing" the P-vortex surfaces, as explained in ref. [12], or else by taking the naive estimate of the number of vortices piercing a plane (per unit area in lattice units), extracted directly from the string tension (see, e.g. ref. [21])

$$f = \frac{1}{2}(1 - e^{-\sigma})\,. \tag{4.2}$$

The two estimates agree fairly well [12], and for $\beta = 2.3$ we find $f = 0.063$. This implies an average distance of $f^{-1/2} \approx 4$ lattice spacings between the centers of vortex cores piercing a plane. Since we have already estimated the vortex thickness at $\beta = 2.3$ to be about 6 lattice spacings, its clear that there must be a substantial overlap between vortex cores, even on a very large lattice. There is nothing in principle wrong with that; vortex cores are not impenetrable objects, and their long-range effects are associated with Dirac 3-volumes, rather than the detailed structure of the core. Our findings for vortex thickness and separation simply indicate, in accordance with some old ideas of the Copenhagen group (Nielsen, Olesen, and Ambjørn in ref. [1]), that the QCD vacuum is more like a liquid of vortices than a dilute gas. Perhaps this is natural for objects which are both condensed and disordered.

4.1 An alternate interpretation

It could also be argued that copy-number dependence is unrelated to the size of vortices in comparison to lattice extension, but is related only to the number of lattice degrees of freedom; i.e. the lattice volume in lattice units. Presumably the number of local maxima of R increases very rapidly with lattice volume. Perhaps one must therefore also increase N_{copy} exponentially with lattice size, in order to see the same drop in projected string tension as observed by BKPV on smaller lattices.

One strong argument against this possibility is the fact that copy number dependence (and violation of center dominance) is substantially greater at $\beta = 2.5$, on a 16^4 lattice, as compared to $\beta = 2.3$, also on a 16^4 lattice. An important difference between these two cases, which in our view is the likely origin of the differing copy-number dependence on these equal-size lattices, is the ratio of vortex thickness to lattice length.

However, let us follow the alternate interpretation, and see where it leads. Specifically, let us suppose that

1. Configurations obtained from over-relaxation are generally, in the spin-system analogy, "false vacua", associated with local maxima of the gauge-fixing functional R. On a large lattice, center dominance (and the other criteria we have mentioned in section 1) are satisfied in almost all of these states.

2. In the true vacuum, associated with the global maximum of R, center dominance is violated.

3. The number of gauge copies required, in order to find a particular copy in which center dominance is violated appreciably, increases exponentially with the lattice volume. Cooling via over-relaxation almost always takes the analog spin system to a metastable, super-cooled state, whose projected configuration satisfies center dominance.

Under these assumptions, consider what happens in the lattice volume $V \to \infty$ limit. Apart from a set of measure zero, *all* random gauge copies, treated via over-relaxation, would fall into one of the "super-cooled" states, which all lead to the same center-projected expectation values. In effect, these metastable states are stabilized in the large volume limit.

If this is the way things work, then it is good enough for the purpose of vortex-finding. It means that on a large but finite lattice the super-cooled states, having the appropriate vortex-finding property, can be identified unambiguously by our procedure. These metastable states are obtained even if they are not the ground state of the analog spin system — and even if they are far from the zero-temperature ground state — on a lattice of large but finite volume.

5. Conclusions

We have found that center-projected lattices are more sensitive to finite size effects than are unprojected lattices, and precision results for center-projected Creutz ratios require lattice sizes which are large compared to one fermi. If accurate numbers (rather than just qualitative results) are required, then center-projected data from numerical simulations must be tested for convergence with respect to: (i) increasing the lattice size; (ii) increasing the number N_{copy} of gauge copies used for center gauge fixing, and (iii) strengthening the gauge-fixing convergence criterion.

Our work was stimulated by the recent findings of BKPV [16], who have demonstrated a significant N_{copy}-dependence for center-projected Creutz ratios obtained on relatively small lattice volumes. In the large N_{copy} limit, a large deviation ($\approx 30\%$) was found between the string tension on unprojected and projected lattices. Our result in the present article is that this N_{copy} dependence is greatly reduced as lattice size increases, and center projected Creutz ratios $\chi_{\text{cp}}(I,I)$ appear to converge to values which are quite close to the asymptotic string tension obtained on unprojected lattices.

It is difficult to say whether the full and center-projected string tensions are in precise agreement in the infinite volume limit. To check such agreement, what is really required is an extrapolation to a triple limit: volume $\to \infty$, $N_{\text{copy}} \to \infty$, and $\delta \to 0$; our data is not yet adequate to extrapolate to this triple limit systematically. But we note again that on the largest lattices we have used, and with results extrapolated (following BKPV) to the $N_{\text{copy}} \to \infty$ limit, the center projected Creutz ratios lie quite close to the asymptotic string tension, as illustrated in figure 4 above. Moreover, on the same large lattices and $N_{\text{copy}} \to \infty$ extrapolation, there is good evidence for asymptotic scaling of the vortex density at couplings $\beta > 2.2$.

We are also unsure of whether the $N_{\text{copy}} \to \infty$ extrapolation actually converges to the values obtained at the global maximum of R. In particular, there is as yet no systematic comparision of results obtained from random gauge copies fixed directly to maximal center gauge, versus gauge copies fixed first to Landau gauge and then to maximal center gauge, in the case of large ($N_{\text{copy}} \sim 20$) copy numbers.[3] However, as emphasized in the previous section, it is not strictly necessary that the local maxima arrived at by our over-relaxation procedure are, from the point of view of center

[3] In fact there are indications, from runs with smaller numbers of gauge copies, that Landau gauge preconditioning obtains a slightly better maximum of R.

projection, essentially identical to the global maximum. If the local maxima obtained by over-relaxation are, instead, super-cooled states (in the spin-system analogy) that are far from the true ground state, then that is perfectly acceptable, providing that the procedure for arriving at such states is unambiguous.

Center projection in maximal center gauge is intended as a method for locating center vortices, on unprojected lattices, from the position of P-vortices on projected lattices. The correspondence of P-vortices with physical objects depends on

1. Scaling of the P-vortex density;

2. Correlation of P-vortex locations on the projected lattice with gauge-invariant observables on the unprojected lattice.

In particular, it is the correlation of P-vortices with unprojected Wilson loops, and specifically the ratios of "vortex-limited" Wilson loops $W_n(C)/W_0(C)$ on the unprojected lattice, which indicate that P-vortices correspond to center vortices (rather than to some other type of object) on the unprojected lattice [5]. Moreover, if P-vortices locate center vortices, and if center vortices produce the full asymptotic string tension, then we must find

(3) Center dominance; i.e. the equality of the string tension on projected and unprojected lattices.

The numerical simulations reported here were concerned with the scaling of the P-vortex density, and with center dominance. From the results of those simulations, we conclude that center projection in maximal center gauge remains a viable method of locating center vortices on full, unprojected lattices, and that these vortices probably account for the entire asymptotic string tension.

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